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On an Erdös theorem on the diameter and minimum degree of a graph

Denote by $G = (V, E)$ a simple connected graph such that every edge has length equal to 1. Here $V = V(G)$ denotes the set of vertices of G and $E = E(G)$ the set of edges of G. The degree of $v \in V(G)$ is the number of edges incident to the vertex and is denoted $\deg(v)$. The maximum degree of a graph G, denoted by Δ, is defined as $\Delta := \text{máx}\{\deg(v) \mid v \in V(G)\}$. Similarly, the minimum degree of a graph G, denoted by δ_0, is defined as $\delta_0 := \text{mín}\{\deg(v) \mid v \in V(G)\}$. The diameter of a graph is defined as $\text{diam}(G) := \text{máx}\{d(x, y) \mid (x, y) \in G\}$. In this work, we obtain good upper bounds for the diamteter of any graph in terms of its minimum degree and its order, improving a classical theorem due to Erdös, P., Pach, J., Pollack, R. and Tuza, Z. (see [1]). On the other hand, we deal with hyperbolic graphs in the Gromov sense. If X is a geodesic metric space and $x_1, x_2, x_3 \in X$, a geodesic triangle $T = \{x_1, x_2, x_3\}$ is the union of the three geodesics $[x_1x_2]$, $[x_2x_3]$ and $[x_3x_1]$ in X. The space X is δ-hyperbolic in the Gromov sense if any side of T is contained in a δ-neighborhood of the union of the two other sides, for every geodesic triangle T in X. If X is hyperbolic, we denote by $\delta(X)$ the sharp hyperbolicity constant of X, i.e. $\delta(X) = \text{ínf}\{\delta \geqslant 0 : \quad X \text{ is } \delta\text{-hyperbolic}\}$. To compute the hyperbolicity constant is a very hard problem. Then it is natural to try to bound the hyperbolicity constant in terms of some parameters of the graph. Let $\mathcal{H}(n, \delta_0)$ be the set of graphs G with n vertices and minimum degree δ_0, and $\mathcal{J}(n, \Delta)$ be the set of graphs G with n vertices and maximum degree Δ. In this work we estimate $a(n, \delta_0) := \text{mín}\{\delta(G) \mid G \in \mathcal{H}(n, \delta_0)\}$, $b(n, \delta_0) := \text{máx}\{\delta(G) \mid G \in \mathcal{H}(n, \delta_0)\}$, $\alpha(n, \Delta) := \text{mín}\{\delta(G) \mid G \in \mathcal{J}(n, \Delta)\}$ and $\beta(n, \Delta) := \text{máx}\{\delta(G) \mid G \in \mathcal{J}(n, \Delta)\}$. In particular, we compute the precise value of $a(n, \delta_0)$, $\alpha(n, \Delta)$ and $\beta(n, \Delta)$ for all values of n, δ_0 and Δ, respectively.

Referencias

[1] Erdös, P., Pach, J., Pollack, R. and Tuza, Z., Radius, Diameter and Minimum Degree, *Journal of Combinatorial Theory* **47** (1989), 73-79.